Absolute interferometry with a 670-nm external cavity diode laser

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In the past few years there has been much interest in use of tunable diode lasers for absolute interferometry. Here we report on use of an external cavity diode laser operating in the visible (λ ~ 670 nm) for absolute distance measurements. Under laboratory conditions we achieve better than 1-μm standard uncertainty in distance measurements over a range of 5 m, but significantly larger uncertainties will probably be more typical of shop-floor measurements where conditions are far from ideal. We analyze the primary sources of uncertainty limiting the performance of wavelength-sweeping methods for absolute interferometry, and we discuss how errors can be minimized. Many errors are greatly magnified when the wavelength sweeping technique is used; sources of error that are normally relevant only at the nanometer level when standard interferometric techniques are used may be significant here for measurements at the micrometer level.

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1. Introduction
Absolute distance interferometry (ADI) refers to use of interferometric techniques for determining the position of an object without the necessity of measuring continuous displacements between points. Whereas a conventional interferometer measures the displacement of a moving retroreflector, an absolute interferometer directly measures the distance to the reflector. Absolute interferometry is of much interest for measurement of large parts (such as in the aircraft industry) or in other applications where it is difficult to generate a continuous displacement of a reflector between the points to be measured.

Various techniques for performing absolute measurements have been discussed in the literature. The method we employ here—wavelength sweeping or wavelength shifting—is not new (dating back at least to 1983), but the technique was not studied extensively until more recently when tunable diode lasers replaced the cumbersome dye lasers originally employed as a light source. Of particular interest are external cavity diode lasers with large tuning ranges that have been employed for ADI.

In our laboratory we have been investigating the suitability of visible (670-nm) diode lasers for wavelength-sweeping interferometry. We have reported preliminary results elsewhere. Here we summarize the results of our research and discuss sources of error that are characteristic of ADI measurements.

2. Wavelength-Sweeping Absolute Interferometry
In a conventional Michelson interferometer used for displacement measurement, the laser wavelength is held fixed and the interferometer system counts fringes as a retroreflector or mirror is displaced. By contrast, the wavelength-sweeping ADI counts fringes as the wavelength of the laser is changed in a continuous manner, while the length of the interferometer arms is held fixed. When the wavelength is changed, the relative phase change in light traversing the two arms of the interferometer (or equivalently, the number of fringes passing the detector) is proportional to the product of the difference in length of the two interferometer arms and the amount by which the frequency of the laser source is shifted.

There are two basic approaches to wavelength-sweeping interferometry. The first is to count fringes as the laser frequency is swept through a known frequency change. For example, the laser can be swept between two well-known atomic absorption lines. The second method, which we em-
ploy here, is to count fringes passing in two separate interferometers, a reference interferometer of known length \( l_{\text{ref}} \) and a measuring interferometer with unknown length \( l_{\text{meas}} \). When a reference interferometer is employed, it is not necessary to know the shift in frequency of the laser source (although the frequency shift could be calculated, if desired, from the observed phase change in the reference interferometer).

If \( l_{\text{ref}} \) and \( l_{\text{meas}} \) are defined as the one-way path-length difference in the two arms of the reference and measurement interferometers, and if the number of fringes passing in the two interferometers are more precisely defined as the observed changes in the phase of the optical interference \( \Delta \phi_{\text{meas}} \) and \( \Delta \phi_{\text{ref}} \), then the unknown length of the measuring interferometer can be determined from

\[
l_{\text{meas}} = l_{\text{ref}} \frac{\Delta \phi_{\text{meas}}}{\Delta \phi_{\text{ref}}},
\]

(1)

reflecting the simple fact that phase change is proportional to the path difference.

To use Eq. (1) it is necessary to accurately know the length of the reference interferometer. We can calibrate this length most easily by inverting the usual measurement process, inferring \( l_{\text{ref}} \) from Eq. (1) by measuring a known distance \( l_{\text{meas}} \). To do this, an incremental interferometer (that is, a displacement-measuring interferometer of standard design) is set up back to back with the moving retroreflector in the measurement interferometer of the ADI. The absolute interferometer is then used to measure the ratio \( l_{\text{meas}}/l_{\text{ref}} \) at two positions, \( l_{\text{meas}} = l_0 \) and \( l_{\text{meas}} = l_1 = l_0 + \Delta l \), where \( \Delta l \) can be measured accurately by the incremental interferometer. The reference length is then given by

\[
l_{\text{ref}} = \Delta l/[\{(\Delta \phi_{\text{meas}}/\Delta \phi_{\text{ref}})_{l_0} - (\Delta \phi_{\text{meas}}/\Delta \phi_{\text{ref}})_{l_1}\}],
\]

(2)

where \( (\Delta \phi_{\text{meas}}/\Delta \phi_{\text{ref}})_{l_0} \) and \( (\Delta \phi_{\text{meas}}/\Delta \phi_{\text{ref}})_{l_1} \) are ratios measured at the two positions \( l_0 \) and \( l_1 \).

This calibration process can be performed with small uncertainty if there is no Abbe offset between the two interferometers (that is, if both measure along the same line). The calibration should be particularly easy and accurate if a standard incremental interferometer shares the same optics with the collinear measuring interferometer of an absolute system. In fact, commercially available laser-tracker systems are often configured in this manner in which a standard incremental interferometer may be present in addition to the absolute system because of its greater speed and accuracy. (The absolute system is then used only to reestablish the measurement position after beam interruption.) In this case, recalibration of the reference length should be trivial, but there are still potential advantages to one employing a reference interferometer with good long-term stability. If the reference is dimensionally stable, its length sets the basic measurement scale independent of the laser wavelength. It is therefore not necessary to employ additional instrumentation to determine the index of refraction of air as would otherwise be required in a normal displacement-measuring system. However, both types of interferometer systems may be subject to errors arising from gradients in the index of refraction along the measurement path—primarily a consequence of temperature gradients—unless these gradients are measured directly along the entire path and appropriate corrections are made.

Before we continue, it is worthwhile to address several issues of nomenclature. The reader should be wary of potential confusion arising from the fact that we are describing systems that include as many as three interferometers—the reference interferometer and measurement interferometer of the ADI and a third interferometer, a displacement-measuring interferometer, external to the ADI. This external interferometer, sometimes referred to as a standard interferometer or incremental interferometer, is a typical fringe-counting Michelson interferometer of standard design.

In discussions of absolute interferometry it is customary to talk about the synthetic wavelength \( \lambda_s \) defined by

\[
1/\lambda_s = |1/\lambda_i - 1/\lambda_f| \approx \Delta \lambda/\lambda^2,
\]

(3)

where \( \lambda_i \) and \( \lambda_f \) are the laser wavelengths at the beginning and end of the sweep, \( \Delta \lambda = |\lambda_i - \lambda_f| \), and \( \Delta \lambda \) is small so that \( \lambda_i \approx \lambda_f = \lambda \). \( \lambda_s \) is an effective wavelength that plays a role analogous to that of the physical wavelength in incremental interferometry. Because of the limited continuous tuning range of diode lasers, the synthetic wavelength is typically several orders of magnitude larger than the physical wavelength. With a tuning range \( \Delta \lambda/\lambda \approx 10^{-3} \), which is typical of older ADI systems, the synthetic wavelength \( \lambda_s \approx 1000 \lambda \). The long synthetic wavelength is indicative of the low resolution often associated with ADI.

### 3. System Description

Our ADI system is shown schematically in Fig. 1. As mentioned in the figure caption, some details of the optics are omitted for clarity in the figure. The primary components of interest are (a) a tunable diode laser; (b) two interferometers, a fixed-length reference interferometer and a measurement interferometer with a movable cube-corner retroreflector in one arm; and (c) optics and electronics for bidirectional fringe counting. These components are described in detail below.

#### A. Laser Source

The laser source is a 670-nm external cavity diode in a Littman–Metcalf configuration. We used two such lasers that give somewhat comparable results. The first, built by the Time and Frequency Division of the National Institute of Standards and Technology (NIST), has been described in a previous publication. Currently we are using a commercial laser that can be tuned over an 8-nm range in 1 s.
B. Interferometer Optics

We use a heterodyne scheme for fringe counting. To do this, the output of the laser passes through an acousto-optic modulator (AOM) that frequency shifts approximately half of the beam energy, generating an 80-MHz frequency-shifted beam at a small angle relative to the unshifted beam. Wedge prisms are used to adjust the frequency-shifted beam parallel to the unshifted beam, and a half-wave plate rotates the plane of polarization of one beam by 90° before the beams pass through a nonpolarizing beam splitter and enter the two interferometers.

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A polarizing beam-splitter cube is used to separate the frequency-shifted beam from the unshifted beam. The geometry is similar to that of a standard heterodyne interferometer, except for the fact that the frequency-shifted beam does not overlap in space with the unshifted beam. Wedge prisms are used to adjust the frequency-shifted beam parallel to the unshifted beam, and a half-wave plate rotates the plane of polarization of one beam by 90° before the beams pass through a nonpolarizing beam splitter and enter the two interferometers.

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to the counters. The differences are constant only if the two pulse trains are exactly in phase. If they are not in phase, the differences will fluctuate by one count depending on whether we happen to read the counters right after a reference channel count arrives or right after a measurement channel count arrives. The relative phases of the pulse trains are recovered by our averaging $(n_1 - n_{\text{ref}})$ and $(n_2 - n_{\text{ref}})$. We find that the effective resolution of the interpolation improves in proportion to the square root of the number of readings, as would be expected if the repeated readings occur at random points along the pulse trains. We typically average 260 readings, which increases the effective resolution of the interferometer from $\lambda/4$ to $\lambda/4(4260) \approx \lambda/64$. This multiple-reading method of fringe interpolation probably has some additional benefits in that it averages out the effects of fast vibrations, which have a potentially serious effect on wavelength-sweeping ADI.

D. Use of ADI as a Displacement-Measuring Interferometer

Our ADI system can be used equally well as an incremental, displacement-measuring system. In practice it is highly desirable to have the option of operating in this mode because accurate ADI distance measurements require at least several seconds of averaging. A practical system would use ADI to establish a zero point and then switch to more rapid data acquisition in the incremental mode. We carried out several tests providing a proof in principle of this mode of operation, but more study would be worthwhile.

The primary difficulty in our using the ADI as an incremental interferometer is that the wavelength is not known a priori. Unlike a standard interferometer in which the length scale is embodied in the vacuum wavelength of the laser as modified by the index of refraction of air, here the basic length scale is the length of the reference interferometer cavity. With the ADI, the wavelength in air of the laser must be determined every time the laser is turned on; luckily, this operation is fairly straightforward to carry out and requires no additional equipment (although it does require additional programming of a computer control). The wavelength can be determined in three steps: (1) measure the absolute position of the measurement interferometer retroreflector, (2) count fringes while the retroreflector is moved to an arbitrary second position, and (3) measure the absolute distance with the retroreflector in the second position. The displaced distance can be calculated from the change in absolute position and, knowing the number of fringes counted during the displacement, we can determine the wavelength. Subsequent changes in the wavelength of this unstabilized laser are not a problem because these changes are automatically tracked. Dead-path issues that would normally complicate use of an unstabilized laser for length measurement do not present any difficulties here because the dead path is always known with micrometer uncertainty.

It is interesting to compare this interferometer with a more conventional system, a standard interferometer employing a wavelength tracker to measure changes in the index of refraction. When a wavelength tracker is used, the initial index of refraction must be determined and the tracker compensates for subsequent changes in the index. With the ADI, the initial wavelength must be determined, and the reference cavity then compensates for changes in air and wavelength arising from either changes in the index of refraction or changes in vacuum wavelength.

4. System Performance

Under metrology laboratory conditions, system performance is entirely satisfactory. We test the performance by setting up the absolute interferometer back to back with a commercial incremental interferometer. It is important to note that we can test performance of the ADI with an uncertainty that is limited primarily by the random fluctuations of the ADI reading. We have carried out previous, extensive studies of uncertainties that arise in testing interferometer systems. These studies give us confidence that we can test interferometer systems with a relative uncertainty of $\Delta I/I = 5 \times 10^{-8}$, or 0.25 $\mu$m at 5 m. (This is the type B combined standard uncertainty, not an expanded uncertainty; here the coverage factor $k = 1$.)

The uncertainty arises almost exclusively from uncertainty in the index of refraction of air; other sources of uncertainty in the test (misalignment of one interferometer relative to the other, Abbe errors, mechanical instability of the test bed, uncertainty in the vacuum wavelength of the commercial laser, fringe interpolation errors of the commercial interferometer, and nonsimultaneous acquisition of the two interferometer readings) can be made much smaller than the refractive index uncertainty if considerable care is taken with the testing procedure. We determine the index of refraction by measuring air pressure, temperature, and humidity and calculating the index of refraction from a modified version of the Edlen formula. The $5 \times 10^{-8}$ relative standard uncertainty arises primarily from uncertainty in sensor calibrations and from uncertainty in temperature gradients along the measurement path, with smaller contributions arising from uncertainties in $\mathrm{CO}_2$ concentration, the possible presence of atmospheric contaminants, and uncertainty in the Edlen equation.

Figure 3 shows the result of a comparison between the ADI system and our incremental interferometer, in a carefully controlled environment, over a range of 0.3–5 m. We interpret any difference in displacements as measured by the two interferometers as an error in the ADI system because the differences shown in Fig. 3 are much larger than the uncertainties described in the preceding paragraph. The sample standard deviation of these results is 0.7 $\mu$m. In more detail, we typically see a standard deviation of 0.3 $\mu$m at distances under 350 mm, 0.6 $\mu$m at 3 m, and 0.8 $\mu$m at 5 m. The best-fit slope of this data, $3.5 \times 10^{-8}$, is small relative to the size of the fluctu-
for the index of refraction of air, which will result in an incorrect value for the length of the ADI reference arm, result in length-proportional uncertainties in distance measurements. As discussed above, the relative standard uncertainty arising from our measurement of the air refractive index is only $5 \times 10^{-8}$; this corresponds to an uncertainty of 0.25 μm at 5 m, small relative to the observed errors. However, it is important to point out that the length of the reference arm may not have good temporal and thermal stability. This important source of uncertainty does not appear in the data shown here because the length of the reference arm was calibrated on the same day that the interferometer was tested, with only small variations in temperature between calibration and testing. If the mechanical integrity of the reference arm is not good, its length may vary with time because of changing temperature or mechanical creep. Furthermore, the geometry of our reference interferometer is somewhat prone to alignment errors and drifting zero offsets, as explained in Section 5. We see day-to-day variations in the apparent length of the reference arm of the order of $\Delta l/l = 1 \times 10^{-6}$.

(3) Errors are a function of turbulence and vibration. In real-world situations, turbulence and vibration would limit the accuracy of the system unless additional measures are taken to compensate for resulting changes in the optical path. The data shown in Fig. 3 were taken with the beam path shielded to reduce turbulence, but even here turbulence is one of the major sources of error in the results. These results are as much as a factor of 4 worse if the beam path is uncovered, even in the relatively benign environment of our laboratory.

(4) Errors can arise from thermal gradients, which give rise to gradients in the index of refraction. Temperature variations can be reduced below 0.07 °C over a 5-m covered path in the carefully controlled conditions of our laboratory. Furthermore, it is possible to measure temperature gradients directly and apply appropriate corrections with an estimated type B uncertainty of 0.03 °C. Uncertainty in temperature variations along the measurement path on the shop floor are much larger, often a major source error. In passing we note that it is the group index of refraction 21 rather than the phase index that is the relevant parameter for ADI.

5. Discussion of Measurement Uncertainties

A. General Considerations

In many respects the sources of error of the ADI are similar to those of any interferometer, but unique aspects of ADI systems must be taken into account. First, the role of the reference interferometer must be considered. The apparent length of the reference must have good long-term reproducibility. In the following subsections we discuss several factors that may cause shifts in the zero position of the interferometer—errors that are independent of measured lengths. Such zero shifts can cause a change in the apparent length of the reference and hence give
rise to a scale error (a length-proportional error) in the length measurement. Zero shifts that vary only slowly over time usually have negligible consequences for incremental interferometry, but they are important here.

Conversely, use of a reference interferometer will reduce the significance of certain common-mode errors. Factors that produce a scale error that is common to the reference and measuring interferometers will have no effect on a length measurement because this method literally measures the ratio of two lengths. Furthermore, scale errors in either interferometer that are truly constant in time—and hence present at the time that the reference length is calibrated—will generate compensating errors in the apparent length of the reference and hence will be inconsequential to subsequent length measurements.

Another unique aspect of the ADI system is that the magnitude of certain errors can be much greater for the ADI than for incremental interferometers. The basic difficulty of the ADI method is that the number of fringes that are counted is small. Consider, as a worst-case example, an ADI system in which the wavelength is swept by \( \Delta \lambda / \lambda = 1 \times 10^{-3} \), or \( \lambda_c = 1000 \times \lambda \). (This value is typical of many ADI systems that have been used in the past; our actual synthetic wavelength is much shorter.) With this ADI, the number of fringes counted (the phase change) when we measure a distance \( d = 1000 \) times smaller than the number of fringes that would be counted with an incremental interferometer if the retroreflector were displaced through the same distance \( d \). Therefore the uncertainty of the phase-change measurement needs to be 1000 times smaller for the ADI to achieve the same uncertainty as an incremental interferometer (at least when measuring short distances in which scale errors are not a great concern). The change in phase is the crucial parameter; any error that is common to the phase at the beginning of the wavelength sweep and the end of the sweep is irrelevant, but an error at only one end of the sweep is magnified by a factor of \( \lambda_c / \lambda \approx \lambda / \Delta \lambda \approx 1000 \) in the final answer. For example, a periodic error in phase interpolation with amplitude \( \pm 3 \) nm, commonly present in incremental interferometers but of negligible importance for many engineering applications, could generate worst-case errors of \( \pm 6 \) \( \mu \)m with \( \lambda / \Delta \lambda = 1000 \) and errors of \( \pm 3 \) nm and \( \mp 3 \) nm at the two ends of the wavelength sweep. For the system we are currently using, \( \lambda / \Delta \lambda = 80 \) and consequently the multiplication of errors is not so severe, but multiplication of errors by more than a factor of 1000 occurs in many ADI measurement systems with a limited wavelength-sweeping range.

Although periodic errors as mentioned above are a significant concern for ADI, a mitigating factor is that averaging can be used effectively to reduce the magnitude of these errors. Interpolation errors are periodic in the phase \( \varphi \) of Eq. (1). (Equivalently, they are periodic in length with period \( \lambda / 2 \).) As a consequence, uniform averaging over a \( 2\pi \) range in \( \varphi \) (one fringe) will eliminate the error.

In many wavelength-sweeping systems it is probable that the wavelength sweep is sufficiently irregular so that \( \varphi \) varies by much more than \( 2\pi \) at the end of each sweep (except during measurement of short distances). The values of \( \varphi \) modulo \( 2\pi \) are then randomly distributed from one sweep to the next, and the results that are average for \( n \) sweeps are expected to reduce periodic interpolation errors by a factor \( n^{-1/2} \).

However, in earlier research we encountered one laser source in which the sweep was sufficiently regular so that successful averaging of periodic errors was not automatically guaranteed; in such cases it is necessary to either modulate the amplitude of the wavelength sweep or introduce a dither in path length. With our current system there is no advantage in doing this, but in the past we achieved significantly better results by introducing a path-length dither into the fixed arms of the measuring and reference interferometers. The technique is practical because the dither need not be as large as the synthetic wavelength; for many sources of error the physical wavelength, not the much longer synthetic wavelength, is the relevant scale over which averaging occurs. (Some reflection errors are strictly periodic at the synthetic wavelength but nearly periodic at the physical wavelength.) With a dither and averaging, we achieved an effective phase measurement uncertainty better than 11 mrad; that is, to attain our observed accuracy with a single sweep would require that the phase measurement uncertainties be reduced to <11 mrad (or expressed as an equivalent length, 600 pm). \(^3\) Even better performance should be possible if the dither is carefully controlled in amplitude and linearity or other appropriate measures are taken to ensure that all phases \( \varphi \) are sampled equally. Gürsel\(^2\) has reported remarkably small phase interpolation uncertainties when cyclic averaging is used to reduce periodic errors.

The following subsections provide a discussion of sources of error that may be expected to be important in an ADI system. In reading this discussion, keep in mind that we have already mentioned the principle sources of uncertainty for our particular interferometer design in Section 4. In the following subsections we discuss both these errors that are clearly an issue with our system and other potential errors that we have not observed directly. The discussion is intended to be sufficiently general in that it includes most sources of error that can arise in ADI measurements regardless of system details; some sources of uncertainty that have negligible impact in our system may be more important when other designs are used.

### B. Turbulence and Vibration

Turbulence and vibration are major concerns with an ADI system. With our current system, \( \lambda / \lambda \approx 80 \), any variation \( \Delta \lambda \) in the optical path that occurs during a single sweep of the laser will show up as a fluctuation \( 80 \times \Delta \lambda \) in the computed length. With an unshielded path, even under laboratory conditions, turbulence is by far the dominant source of error.
unless long averaging times are used to reduce these random errors. Vibration is less of a problem than turbulence in situations we have encountered thus far. Because we make multiple readings of the phase at the end of each sweep, the effect of vibrations is diminished through averaging, even if the measurement is a single sweep.

The performance of our current ADI system in realistic environments is turbulence limited. We can predict performance by measuring the turbulence and vibration using a commercial incremental interferometer. All that is required is to measure the displacement of a nominally stationary retroreflector as a function of time and compute the root-mean-square (rms) change in position as a function of the time interval between readings. Figure 4 shows typical results for our laboratory, at a distance of 5 m with no shielding of the beam path. If the diode laser sweeps through its frequency range in a time interval $t$, then the rms fluctuation in position at time $t$ on the graph, multiplied by $\lambda/\lambda_s$, is the predicted standard uncertainty for a single-sweep determination of the length by use of the ADI. This calculation gives good semiquantitative predictions for observed fluctuations in ADI readings. We find that the calculation slightly overestimates turbulence errors, presumably because global pressure fluctuations that are common to reference and measurement interferometers will not generate fluctuations in the ADI but will affect the incremental interferometer reading.

For the data shown in Fig. 4, the fluctuations scale roughly as $t^{1/2}$; if the amplitude of the laser frequency sweep is held constant, increasing the sweep frequency should decrease the effect of turbulence in a predictable manner. Thus one method to reduce turbulence errors is to employ a fast-sweeping laser. An alternative is to add a fixed-wavelength laser to the system. If the fixed-wavelength laser shares the same optics as the measuring interferometer, it can be used to correct for variations in the optical path that occur during a sweep as a result of turbulence or vibration. The addition of a fixed-wavelength laser complicates the system but should provide significant benefits when a slow-sweeping laser is used.

C. Scale Errors

Several effects can give rise to a length-proportional error (a scale error). For example, a small uncertainty arises in calibration of the reference length, as discussed in Section 4. Of greater concern are potential variations in the length of the reference after it is calibrated. Variations can arise from both purely mechanical sources or from optical effects. Thermal or stress-related changes in a poorly constructed reference may generate either reversible or irreversible changes in the reference length. Even for a perfectly constructed reference interferometer, on long time scales it is necessary to take into account temporal instability of materials that are used to set the reference length in the interferometer. We have not yet studied the long-term stability of our reference cavity, but we know that, at a minimum, the fractional length will change by $\Delta l/\ell = 6 \times 10^{-7}$ for each 1 °C change in temperature because of thermal expansion of the fused-silica spacer.

An additional scale error will occur because the reference interferometer compensates only for variations in index of refraction if atmospheric conditions are the same in the measurement path and in the reference. Gradients in atmospheric temperature or pressure will not be compensated correctly.

D. Electronics

Errors in the electronic phase meter are a potential concern. For our method of measuring phase, the predominant expected error is the effective resolution of our system, $\lambda/64$ for a single sweep (or $\lambda/140$ for typical data where we take five sweep averages). This puts a lower limit on the attainable uncertainty. To determine a length, at least four phase measurements are required—the phases at the beginning and end of the sweep for both the measurement and the reference interferometers. For each of these measurements the standard uncertainty arising from $\lambda/64$ resolution is $(\lambda/64)/(2\sqrt{3})$, where we assume a uniform distribution of errors over the resolution interval. At any given length the resulting uncertainty in the measured result can be determined by our propagating the phase measurement uncertainties in Eq. (1). We observe uncertainties that are close to this resolution limit when measuring short distances with a shielded beam path.

Other electronic phase measurement errors are not expected to be as large as the resolution error. There is some danger that electrical cross talk from the high-level AOM drive signal penetrates into the measurement channels and corrupts the phase measurement, but the problem is not severe; it would require a cross-talk signal whose amplitude is 20% of our true signal (hence easily observable) before the error would be comparable with our effective single-
sweep resolution limit ($\lambda/64$). Most phase measurement errors are periodic in phase and hence will be reduced by our averaging over multiple sweeps.

E. Ghost Reflections into the Detector and Optical Mixing

Although it is not one of the most serious problems, optical mixing and ghost reflections are of some concern in ADI because these small errors are magnified by $\lambda/\Delta\lambda$. Optical mixing and reflections are similar in effect; in both cases a portion of the light beam travels along an unintended path to the detector, where it perturbs the phase of the detected electric field.

In a heterodyne interferometer such as used here, distance information is encoded as a phase shift $\varphi$ between two beam components with orthogonal polarizations and slightly different frequencies $f_1$ and $f_2$. Ideally, one beam component traverses one arm of the interferometer while the other component traverses the other arm, so that the phase difference between the recombined beams is $\varphi = (f_1 - f_2)\delta t + 4\pi/\lambda$ where the spatial phase variation depends on the difference in length of the two arms ($l$). The formula is not valid if optical mixing or ghost reflections cause a small fraction of the light to travel an unintended path; optical mixing causes some light to travel down the wrong arm of the interferometer, and reflections might cause small beam components to travel along a number of different paths. Under these circumstances, the phase of one or both frequency components are corrupted as indicated schematically in Fig. 5; the electric field $E_f$ of the beam with frequency $f$ is the vector sum of a desired component with amplitude $E$ and a spurious component with amplitude $\varepsilon$. The spurious component has traveled a different path and consequently is shifted in phase by an angle $\alpha$. This phase angle between the two fields changes periodically as a function of the path difference between the desired beam and the spurious beam and as a function of laser wavelength. The perturbation in the phase of the electric field $E_f$ is maximal when $\alpha \approx \pm 90^\circ$, at which point $\varphi$ is in error by an angle $\delta \varphi_{\text{max}}$ with

$$\delta \varphi_{\text{max}} \approx \pm \tan^{-1}(\varepsilon/E) \approx \pm \varepsilon/E.$$  \hspace{1cm} (4)

The error of significance to a wavelength-sweeping ADI measurement is the difference of the two phase errors $\delta \varphi$ at the two ends of the wavelength sweep.

Optical mixing can be a significant problem in standard heterodyne interferometry, but it is not a serious concern here because the frequency components are generated in a manner that gives nearly perfect separation of the two components; and with our interferometer geometry (spatially separated beams), imperfect performance of the polarizing beam splitters does not give rise to optical mixing. An ADI relying on more conventional polarization coding for heterodyne phase detection could have more serious problems, particularly if the wavelength sweep is small so that the multiplication factor $\lambda/\Delta \lambda$ is large.

Reflections are more likely to cause difficulties than is optical mixing, particularly when we use cube beam splitters and solid cube-corner retroreflector, as shown in Fig. 2. The large number of glass–air interfaces nearly perpendicular to the beam present a significant concern. If a specular reflection from an optical element with reflection coefficient $r$ inadvertently enters the detector, then $\varepsilon/E = r^{1/2}$ in approximation (4). Reflections can cause a remarkable variety of problems. The effect of reflections into the detector has been described by Kikuta and Nagata, but our situation is considerably different from theirs and requires further discussion.

Interpolation errors that are periodic with the measured distance will occur if a stationary interface in the measuring arm of the interferometer, oriented approximately normal to the beam, produces a reflection into the detector. This type of reflection is not a serious concern when we use our interferometer geometry but is important when we use a setup in which the beam strikes the center of the retroreflector and retraces its path, such as is shown in Ref. 8. (This situation is called a case A reflection in Ref. 8.) The effect of these reflections is negligible with our interferometer geometry because the return beam is displaced from the outgoing beam. Referring to Fig. 2, a reflection from the surface of the beam splitter at point 1 or point 2 could enter our detector only if the beam-splitter surface happened to be oriented at the correct angle so that the reflection reaches the detector, located along the path of the return beam. The reflected beam will enter the detector at some angle $\beta$ relative to the desired return beam, and consequently the relative phase of the two beams will vary linearly with position across the region where the two beams overlap at the detector. The detected signal is an average over these phase variations, and the averaging reduces reflection errors to insignificant levels in our apparatus for realistic values of $\beta$. After we take into account this averaging over the beam profile, approximation (4) becomes

$$\delta \varphi_{\text{max}} \approx (r^{1/2})\exp[-2(\pi \varepsilon \beta /\lambda)^2].$$  \hspace{1cm} (5)

Approximation (5) was derived under the assumption that the reflected power is small and that all interfering beams are Gaussian with the same waist ($w$).
Also, any possible wave-front spreading and curvature were ignored.

Errors that behave somewhat differently (not a periodic function of distance) can occur as a consequence of reflections off the face of the moving retroreflector (point 3 in Fig. 2), which might occur at unpredictable points depending on the exact orientation of this surface. The retroreflector is typically much farther away from the detector than is the beam splitter; consequently it is less probable that this reflection will enter the detector, but at sufficiently large distances, where \( \beta \) is small, the error is larger. However, for our apparatus, \( \beta \) remains relatively large when we are measuring distances under 5 m (\( \beta > 0.0006 \) and \( \pi \nu \beta / \lambda > 5 \)). The error predicted by approximation (5) approaches zero extremely rapidly when \( \pi \nu \beta / \lambda \) is much larger than 1; here \( \delta \varphi_{\text{max}} < 2 \times 10^{-29} \).

Much more likely to occur are certain errors arising from double reflections. Referring once again to Fig. 2, consider a reflection from the beam returning to the beam splitter from the retroreflector (point 5) followed by a second reflection at point 2. These two reflections from the same surface, with only a retroreflector interposed between the two, will generate a beam accurately parallel to the primary beam, but displaced from the primary beam by a distance \( \approx 4x\gamma \) where \( x \) is the distance to the retroreflector and \( \gamma \) is the angle between the beam-splitter cube surface normal and the laser beam. Unless the surface of the beam splitter is angled sufficiently so that the spurious beam does not overlap the primary beam, the spurious reflection will enter the detector and shift the phase. If \( r \) is the reflection coefficient of the surface, the double reflection will produce maximal phase errors of \( \pm r \) rad in a single phase reading, hence a maximum error of \( \pm 2r \) in the measurement of \( \Delta \varphi \). This error can be avoided if the beam splitter is tilted slightly and measurements are not made with the retroreflector close to the beam splitter. Even if these precautions are not observed, the error can be reduced greatly through dithering and averaging because the error is quasi-periodic with a spatial period equal to the fringe spacing.

Errors from double reflections are generally not of great concern because the doubly reflected power is small. However, the error can be nonnegligible if the wavelength sweep is small and if averaging is not employed. For a 0.5% reflection from a coated surface, the phase error can be as large as \( 2r = 10 \) mrad, which would yield a length error of \( 2r \times (\lambda / 4\pi) = 0.5 \) \( \mu \)m for our worst-case example of \( \lambda / \Delta \lambda = 1000 \). In fact the error could be double this size because a similar effect will occur as a consequence of a reflection at point 6 followed by a reflection at point 1. The two double-reflection errors can add or subtract from each other, depending on the width of the cube and the magnitude of the wavelength sweep.

In passing we note that these double reflections are likely to be present in high-precision standard (incremental) interferometry. Here the retroreflector is typically close to the beam splitter and the beam-splitter cube is aligned accurately nearly perpendicular to the beam so as to reduce optical mixing errors. A beam splitter aligned within 1° (as recommended by the Association of German Engineers\( ^{23} \)) will produce double-reflection errors over the first \( \sim 70 \) mm of travel. With 0.5% antireflection coating, the amplitude of the nonlinearity for a double reflection, \( \pm \nu / 4\pi \), is 0.5 nm peak to peak. The overall nonlinearity will depend on whether reflections from the front face and back add constructively or not; as a consequence, the nonlinearity will vary with temperature, reaching a maximum of 1 nm peak to peak when the optical length of the beam splitter gives constructive interference of the reflections. The situation is potentially worse for interferometers in which additional optical elements such as quarter-wave plates or wedge plates are present in the beam path, causing additional reflections.

Double reflections can also generate zero shifts—errors independent of the measured length. For example, a zero offset can occur from a double reflection in the fixed-length interferometer arm such as a reflection at point 10 in Fig. 2 followed by a reflection at point 7. In fact, this double reflection is likely to occur in a typical interferometer with the fixed-arm retroreflector mounted directly against the beam splitter. In addition, when solid cube-corner retroreflectors are used, other double reflections and resultant zero shifts are almost guaranteed to occur—namely, internal reflections in the two cube corners (a reflection at point 4 followed by a reflection at point 3, or at point 9 followed by point 8). Although the zero offsets arising from these double reflections are independent of the measured length, they are potentially significant because the zero shift will change with time if the two ends of the wavelength sweep, \( \lambda_0 \) and \( \lambda_f \), drift in time.

In the reference interferometer, any of the reflections as described above can change the apparent reference length and hence produce a scale error in ADI measurements. The problem is not severe unless the synthetic wavelength is long; we would expect to see nonnegligible consequences when \( \lambda / \Delta \lambda > 1000 \). Single reflections can be avoided, and double reflections are small if coated optics are employed. The analysis above indicates that a number of double reflections are likely to be present but that each should contribute only \( \sim 0.5-\mu \)m zero shift even with a fairly large synthetic wavelength (\( \lambda / \Delta \lambda = 1000 \)). With our short synthetic wavelength and long reference arm the problem is negligible: For \( \lambda / \Delta \lambda = 80 \) we expect to see zero shifts of only \( \sim 40 \) nm in our 3-m reference arm. If the zero shift varies in time, consequent scale errors will be only \( \Delta l / l \sim \pm 1 \times 10^{-8} \).

Finally, it must be noted that reflections back into the laser are a potential concern in the absence of optical isolation because the reflection might destabilize the laser. If the reflection merely shifts the wavelength in a continuous manner (frequency pulling), it should have no effect on our measurement, but a discontinuous mode hop will degrade performance.
between the images and is approximated to second order in $\Delta u$.

The change in distance measured, approximated to second order in $\Delta u$, is

$$d \sin u(\Delta u) - d \cos u(\Delta u)^2/2$$

where $\delta$ is the distance between the two nodal points measured in a direction perpendicular to the laser beam. For a Michelson interferometer of the standard geometry used for metrology systems, $\delta$ can be interpreted as one half the offset between the two interfering beams at the output of the interferometer; it is ideally zero and almost always less than 1 mm even for a poorly aligned system. With $\delta = \theta = 0$, Eq. (6) reduces to the standard second-order result for an angular misalignment $\Delta \theta$. In our system, where the two incoming beams are offset from each other, $\delta$ is given by one half the offset between the centers of the two incoming beams when the system is aligned correctly. As a consequence of this offset ($\approx 9$ mm offset or $\delta = 4.5$ mm), the first-order term is of significance.

The primary manifestation of this problem is that it increases sensitivity to alignment errors in our reference interferometer. As a result of alignment instability in mirror mounts, the incoming laser beam must be realigned periodically with the reference interferometer. If repeated alignments are carried out with an angle uncertainty $\Delta \theta$, the effective length of the reference will change as indicated by Eq. (6). We determine alignment by looking for a good overlap between the two beams at the detector. After propagation through many optical elements, the beam quality is not ideal at the detector, and it is therefore possible that this overlap could vary by as much as $\pm 0.5$ mm. For our 3-m reference arm, this corresponds to $\Delta \theta = \pm 8 \times 10^{-5}$. The usual, second-order cosine error is entirely inconsequential for our system with $\delta(\Delta \theta)^2/2 = 10$ nm, but a larger first-order error can occur because of the offset; $\delta \Delta \theta = \pm 0.4$ mm, suggesting that variations in the length of the reference might be as large as $\sim 0.8$ mm after realignment. Thus scale errors as large as $\Delta l/l = 2.6 \times 10^{-7}$ might occur as a consequence of varying alignment.

Note that whereas the normal cosine error is a length-proportional scale error, this first-order error is an alignment-dependent zero offset, independent of the measured length. However, the zero offset in our reference interferometer becomes a scale error in actual measurement results.) In standard, incremental interferometry this first-order alignment error would usually not be an issue at all; if the alignment remains constant during a measurement, the measured displacement will not be affected by the zero offset.

G. Dispersive Effects

Dispensive elements can give rise to several kinds of errors. Shifts in zero position, similar to shifts that can occur in a standard interferometer, will occur in the ADI if the center position of the wavelength sweep varies and if dispersive elements are present in only one arm of the interferometer. For example, the wedge prisms in our system will cause a zero shift of approximately 1 mm if the center of the wavelength sweep drifts by 1 nm. The shifts that occur in the ADI differ somewhat from the shifts in a standard interferometer, in as much as the wavelength-dependent variations in group refractive index (not the phase index) give rise to the effect.

Additional errors might be expected as a consequence of alignment changes, synchronous with the
wavelength sweep, that are a consequence of dispersive optical elements in the beam path. When dispersive elements are present, the beam direction, position, and shape may change slightly as the wavelength is swept. These changes may generate a number of small spurious phase shifts that are synchronous with the wavelength sweep and therefore produce length measurement errors magnified by $\lambda/\Delta \lambda$.

A number of optical elements in our system might generate changes in the beam direction with changing wavelength. The AOM is the most obvious example—producing a deflection of the frequency-shifted beam that is proportional to wavelength—but other optical elements such as the beam-expanding telescope or anamorphic prisms might also cause some angular variations. Consider what occurs if the wedge prisms are adjusted to align the frequency-shifted beam parallel with the unshifted beam when the wavelength is at one end of the sweep. At the other end of the sweep the beams will be misaligned by some angle $\Delta \theta$ proportional to $\Delta \lambda$. (In practice it is better to align the frequency-shifted beam so that it is misaligned by equal angles $\pm \Delta \theta/2$ at the two ends of the sweep.) It is clear from the beam geometry that, for finite-width beams that are misaligned relative to each other by $\Delta \theta$, phase shifts of one beam relative to the other must be present, varying linearly across the beam profile and proportional to $\Delta \theta$ (hence proportional to $\Delta \lambda$). The varying phase causes loss of contrast; we observe changes in signal strength by approximately 30% as the wavelength sweeps and produces significant phase shifts across the beam profile. Based on the known wavelength deflection of the AOM, we expect that phase variations across the beam profile are approximately 3 rad.

Phase shifts of this size are capable of producing significant errors in length measurement, but it is important to note that errors that do not vary with time have no practical consequences (unless they are nonlinear functions of the measured distance). Constant zero shifts do not affect the measurement of the distance between any two points in space, and, as mentioned above, a linear, length-proportional error (a scale error) that is constant in time will be compensated automatically by the calibration procedure for determining the length of the reference interferometer. Therefore errors will have a measurable effect only if they vary in response to changes in some other factor. The potential importance of phase shifts that are proportional to $\Delta \lambda$ is diminished because of the fact that, although the phase errors change linearly with $\Delta \lambda$, actual length measurement errors are independent of $\Delta \lambda$, as a consequence of the $\lambda/\Delta \lambda$ multiplication factor. Consequently the length measurement errors should reproduce well on a day-to-day basis even if $\Delta \lambda$ is not constant.

However, the error that arises from the phase shifts described above can vary with changes in the measured position if there is a change in the area of overlap of the two beams interfering at the detector, which will occur if the direction of motion of the retroreflector is not well aligned with the beam. In our case, the error will change by approximately 1.6 $\mu$m for a 1-mm shift in the overlap of the beams. With careful beam alignment (aided by a position-sensitive detector) the shift can be kept below 0.2 mm over our 5-m measuring path, which gives no more than a 0.3-$\mu$m error in our results. It is somewhat more difficult to align our reference arm (as mentioned in Subsection 5.F), and day-to-day alignment variations might cause $\pm 0.8-\mu$m zero shifts in the reference length, in addition to the $\pm 0.4-\mu$m variations described in Subsection 5.F.

The beam geometry also suggests that there must be additional phase shifts associated with the AOM deflection that are approximately proportional to $(\Delta \theta)^2$ and to the distance between the AOM and the plane where the beams are recombined because of a $1/\cos(\Delta \theta)$ dependence of the path length on the angle of misalignment $\Delta \theta$. These phase shifts, quadratic in $\Delta \lambda$, might vary in time, but we estimate that the resulting zero shifts will never exceed 30 nm.

Angular deviations of the frequency-shifted beam will also give rise to lateral shifts in the relative positions of the two beams interfering at the detector. If the wave fronts are not perfectly flat, the change in relative position will give rise to additional phase shifts (which are similar to those phase shifts caused by the angular misalignment of the two beams as described above in this subsection) that can change if beam overlap changes as a function of the measured distance. We estimate that any such errors are unlikely to be as large as the errors already discussed in the preceding paragraphs.

H. Laser Diode Spectral Characteristics

Mode hops are usually fatal to the wavelength-sweeping measurement technique. At a minimum, the mode hop results in a loss of all subfringe phase information, although it should be possible to recover this information with sufficient averaging and appropriate dithering of the wavelength sweep.

We usually employ a wavelength-sweeping interval $\Delta \lambda$ larger than can be attained reliably without mode hops; consequently we must continuously check for the presence of mode hops and periodically realign the laser or adjust the operating parameters to avoid problems. Reflections of the beam off of the two faces of a thin, uncoated glass plate interfere at a photodetector with 10-MHz bandwidth. (In effect this is a simple Fizeau interferometer with a free spectral range much larger than the free spectral range of the diode external cavity.) Mode hops cause a rapid, near-discontinuous change in the photodetector output that is easily distinguishable from normal photocurrent variations if the detector output is passed through a high-pass filter. In addition, mode hops can often be detected by circuitry that checks for successive counts arriving too close in time to each other in the interferometers of the ADI system.

Even in the absence of mode hops, spectral purity of the laser output is often an important concern.
Clearly the measurement will work poorly as the measured distance approaches the coherence length of the laser. Multimode oscillation of the diode can give rise to errors even if a second mode is only weakly excited and produces little loss of contrast in the interference fringes. If the second mode is present at only one end of the sweep and perturbs the measured phase by $\Delta \phi$, the resulting error in distance measurement will be $(\Delta \phi / 4\pi) \lambda_c$. The error varies periodically with the measured distance. If two adjacent longitudinal modes of the laser are excited, the error is equal to the cavity length of the laser. The maximum error $e_{\text{max}}$ depends on the ratio of the power in the secondary mode ($P_{\text{secondary}}$) to the power in the primary mode ($P_{\text{primary}}$):

$$
e_{\text{max}} = \pm (\lambda / 4\pi) \tan^{-1}(P_{\text{secondary}} / P_{\text{primary}}), \quad (7)$$

where the analysis is similar to that discussed in connection with approximation (4). Power, rather than amplitude, is the relevant parameter in Eq. (7) because the beat signal from multimode interference is at frequencies well above the passband of our electronics.

We periodically check for multimode oscillation using a scanning Fabry–Perot spectrum analyzer, but we do not have an on-line, continuous monitor for multimode oscillation. When we use the NIST-built diode laser, indirect evidence strongly suggests that we may have occasionally encountered errors from multimode behavior as the laser drifted toward a condition of mode hop.

Some authors have commented on degradation of performance of ADI systems that is due to the phase noise of the laser. We see no evidence of this for the lasers we used; any phase noise that is present in the system is masked by our limited resolution or by fluctuations arising from residual turbulence that is present in the beam path even when it is covered. The potential problem is that fluctuations in laser wavelength, frequency, and phase occur on a time scale comparable to the delay between measurements made in our reference interferometer and our measurement interferometer. In principle, some delay is an inevitable consequence of the time of flight of light through the apparatus, although longer delays might arise from a nonsimultaneous reading of the fringe counters or from slightly different electrical characteristics of the detectors and amplification circuitry. In any case it is unlikely that electrical delays would exceed $0.5\ \mu s$ for a well-designed system, and time-of-flight delays will be smaller than this value for measurements of distances under 75 m.

Some variations in laser wavelength and frequency are expected simply because we are sweeping the wavelength; the sweep is greatly slowed but does not completely stop during the period of time that readings are acquired at the end of the sweep. At the time when the counters are read, residual sweeping is less than 0.5 nm/s, and the change in laser wavelength during a possible 0.5-\mu s delay should not exceed $\Delta \lambda / \lambda = 4 \times 10^{-10}$. This wavelength variation that is due to residual sweeping is greater than wavelength variations arising from electrical or mechanical fluctuations that are characteristic of this kind of laser under static (no-sweeping) conditions.

At a distance of 10 m, the $4 \times 10^{-10}$ wavelength variation will give rise to a phase variation of only 0.07 rad. This variation is much smaller than the $\lambda/4$ single-reading resolution of our system but comparable to the $\lambda/140$ effective resolution we achieve with five sweep averages. Like resolution errors, the effect of fluctuations from rapid, random phase noise will be reduced by our averaging over multiple readings of our counters at the end of the sweep, and consequently we do not expect to see any degradation of performance because of phase noise. Residual sweeping of the laser could give rise to a nonrandom phase error that might not average to zero. Based on the estimates above, one might imagine that under unfavorable circumstances the phase error could be comparable to our effective resolution limit. This problem can be avoided if some care is taken to ensure that counter readings are acquired symmetrically with respect to the extremes of the sweep so that there is no net average change in wavelength between readings of the measurement channel and readings of the reference channel. In actuality we see no significant changes in the measured length even if readings are not acquired with perfect symmetry, indicating that any errors arising from the residual sweeping can easily be kept small.

1. Summary of Sources of Uncertainty
In the preceding analysis we discussed many sources of uncertainty that may affect ADI systems, but we conclude that many are not particularly important in our system. In summarizing the primary sources of uncertainty in our system, we should distinguish between those errors that dominate the experimental data of Fig. 3 and the additional uncertainties that can be expected during day-to-day operation. Here we recapitulate what was discussed in the preceding sections:

(1) Under carefully controlled conditions, characteristic of the data shown in Fig. 3, the primary sources of uncertainty are
(a) Turbulence: We estimate a standard uncertainty of $0.9 \ \mu \text{m}$ at 5 m based on measurements as described in Subsection 5.B. As explained in Subsection 5.B, the estimate is somewhat too high, slightly overestimating the observed 0.8-\mu m experimental standard deviation of fluctuations observed at 5 m.
(b) Resolution error: We estimate a type B standard uncertainty of $0.3 \ \mu \text{m}$ at 5 m (0.15 \mu m at short distances), calculated as described in Subsection 5.D.
(c) Uncertainty in the length of the reference interferometer: We estimate a type B relative standard uncertainty of $5 \times 10^{-8}$ (or 0.25 \mu m at 5 m) in our measurement results arising primarily from imperfect knowledge of the index of refraction as described in Section 4.
(2) In actual use on the shop floor we can expect much larger uncertainties:
(a) When the beam path is uncovered, turbulence errors can be expected to be at least four times larger than quoted above.
(b) Unmeasured air temperature gradients will introduce length-dependent errors of the order of $\Delta l/l = 1 \times 10^{-6} \Delta T$, where $\Delta T$ is the temperature variation along the beam path, expressed in degrees Celsius.
(c) Thermal and temporal instability of the reference interferometer will introduce errors. The stability of the reference interferometer is not the subject of this study, but we know that, at a minimum, the reference length will vary (because of thermal expansion of the fused-silica spacer) by $\Delta l/l = 6 \times 10^{-7} \Delta T_{r}$, where $\Delta T_{r}$ is the deviation of the reference interferometer temperature from the temperature at which the reference length is calibrated, expressed in degrees Celsius. A reference of more stable construction could still be subject to $\sim 1\mu$m variations in the apparent optical length for reasons described in Subsections 5.F and 5.G.

6. Testing Absolute Distance Interferometry

It should be apparent from the discussion above that a great number of potential errors may affect ADI systems. Many of the most likely troublesome errors occur over a length scale of one synthetic wavelength or less, and consequently common practice is to test the interferometer over this range. This is certainly a reasonable procedure often dictated by experimental necessity, but it may not accurately test interferometer performance over the long ranges where the ADI is most useful.

In principle, errors can occur on a surprising variety of length scales from submicrometer to tens of meters, including laser wavelength, synthetic wavelength, coherence length, Rayleigh range of beam in the interferometer, diode optical length, length of diode external cavity, length of reference interferometer, possible characteristic length scales of atmospheric turbulence, and the synthetic wavelength associated with AOM frequency (80 MHz). For example, multimode oscillation with two adjacent modes lasing gives rise to periodic errors with a spatial period equal to the optical length of the external cavity laser (or conceivably the optical length of the diode itself if it is not well coated). The length dependence of actual measurement errors will be further complicated by the fact that we are really measuring the ratio of two lengths; when the measured length is equal to the reference length certain errors will cancel. Thus it is potentially important to test the laser over a range that may be significantly greater than the synthetic wavelength. In principle, certain errors such as reflection errors discussed above could occur at any position. If it is expected that the ADI will be used at large distances, it is particularly important to test the system near the end of its operating range, where errors from turbulence are greatest, where the greatest problems will be encountered from the laser coherence length and divergence of the laser beam (which reduces signal strength and might increase feedback into the laser), and where the demands on the dynamic range of counting electronics are greatest. We find unexpected degradation in the performance of our system at distances greater than $\sim 10$ m; the cause of this degradation is still being investigated.

7. Conclusions

We achieved accurate results under good conditions, but several possible improvements in the apparatus would increase its utility. The addition of a fixed laser common path with the tunable laser would reduce sensitivity to turbulence. A better reference interferometer would ameliorate various problems that limit the day-to-day reproducibility of the apparent optical length of our current reference. We have performed initial investigations with our reference interferometer replaced by a Zerodur Fabry–Perot cavity. With a Fabry–Perot reference, $\Delta l$ is fixed by one tuning the laser through an integer number of orders of the cavity whose length has been determined in the manner described above. We have carried out a preliminary investigation of this scheme but it is too soon to decide if the Fabry–Perot reference will improve performance of the system.

We thank Michelle Stevens, Leo Hollberg, and Richard Fox of the Time and Frequency Division, NIST, for many helpful discussions and for providing us with an external cavity diode laser.

References and Notes

7. H. Kikuta, K. Iwata, and R. Nagata, “Distance measurement

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10. Certain trade names and company products are mentioned in the text or identified in illustrations for adequate specification of the experimental procedure and equipment used. In no case does such identification imply recommendation or endorsement by NIST nor does it imply that the products are necessarily the best available for the purpose.
15. The group refractive index \( n_g \) is related to the phase refractive index \( n_p \) by \( n_g = n_p - \left( \frac{dn_p}{d\lambda_0} \right) \lambda_0 \), where \( \lambda_0 \) is the vacuum wavelength. See, for example, J. M. Rüeger, Electronic Distance Measurement (Springer-Verlag, Berlin, 1990), pp. 51–52.