Sub-micrometre distance measurements with a broadly tunable short-external-cavity InGaAsP/InP diode laser

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Abstract: Measurement of distance using a custom-designed, broadly tunable InGaAsP/InP short-external-cavity diode laser is described. A tuning range of over 100 nm was achieved with the custom-designed laser in a diffractive optical element short external cavity. This tuning range made it possible to achieve a sub-micrometre resolution in measurement of distance with a single laser source for an interferometer. A non-linear, least squares fitting method was used to extract the displacement from the raw data. This fitting method showed a potential for extraction of accurate displacement in the presence of noise.

1 Introduction

Compact and non-contact measurements of absolute displacement with sub-micron resolution are of interest to many industries and the medical field. In the automotive industry, where machine vision is widely used, a compact, absolute measurement system solves many mechanical and technical issues. Intelligent robotic arms are needed for next generation production lines and will require accurate measurements of displacement.

We present in this work measurement of distance using a broadly tunable short-external-cavity (SXC) InGaAsP/InP diode laser in an optical interferometer. We report absolute distance measurement with sub-micrometre resolution using a single laser source, similar to the resolution obtained using the wavelength multiplexing theory [1–4]. We used custom-designed asymmetric multiple-quantum well (AMQW) lasers [5, 6] that were tuned using a diffractive optical element (DOE) in a-(SXC) configuration [7, 8].

The motivation for this work was to determine the suitability of a single DOE SXC AMQW laser in an optical interferometer for measurement of distance. Optical interferometers have been widely used in many industrial and medical applications. Interferometers are easy to construct and can be accurate in measuring distances. An unambiguous measurement of distance over a wide range of distance can be achieved by tuning the laser source between two or more wavelengths and using the differences of the phases of the interference fringes to extract the difference in path length between the two arms of the interferometer. The benefit of this multi-wavelength method is to eliminate the need to count fringes. In contrast to a fringe counting technique, the multi-wavelength approach allows for an interruption of the measuring sequence. Allowing interruption opens a range of industrial and medical applications.

Two techniques have been published to achieve measurement of displacement without ambiguity over differences in the arms of an interferometer from 1.2 m to 14.5 cm with resolutions better than several micrometres [2, 3]. In this work we report measurement of the difference between arms of an interferometer with sub-micrometre resolution using a single, broadly tunable diode laser. We report the use of a nonlinear least squares fitting technique to extract the displacement (i.e. the difference in path length between the arms of the interferometer) from the raw data and compare this technique with the more common phase subtraction technique.

2 Theory of operation

2.1 Single wavelength ranging

The output of a Michelson interferometer is given by [9]

\[ I(\tau) = (K_1^2 + K_2^2)I_0 \left[ 1 + \frac{2K_1K_2}{K_1^2 + K_2^2} \Re\left\{ \gamma(\tau) \right\} \right] \]

(1)

where \( K_1^2 I_0 \) is the irradiance in one arm of the interferometer, \( K_2^2 I_0 \) is the irradiance in the other arm of the interferometer and \( \gamma(\tau) \) is the complex degree of coherence of the light, with \( |\gamma(\tau)| \leq 1 \). The degree of coherence is a measure of the ability of the light to self-interfere. The fringe visibility of the interferometer is proportional to \( |\gamma(\tau)| \). The argument \( \tau \) is the difference in propagation times for the light in each arm of the interferometer. Sources with narrow line widths, such as external cavity lasers, will have high degrees of coherence and will form fringes for large and small \( \tau \).

In the event that the irradiances in the two arms of the interferometer are equal (i.e. \( K_1 = K_2 \)), and that the light from the source shows perfect coherence (i.e. \( |\gamma(\tau)| = 1 \)), the output of the interferometer is given by the simple equation

\[ I(\tau) = I_0[1 + \cos(\phi)] \]

(2)

where \( \phi(\tau) \) is the phase difference between the measurement

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arm and the reference arm and can be written as

$$\phi(\tau) = \frac{2\pi}{\lambda} (S_1 - S_2) = \frac{4\pi S}{\lambda}$$

(3)

$S$ is the difference of the path lengths of the two arms, or the displacement, of the interferometer, and $\lambda$ is the wavelength in air of the light being used and $\tau = 2\pi n/c$ with $c/n$ the phase velocity of the light in the arms. Equations (2) and (3) demonstrate that the phase difference is a function of the wavelength and that measurement of displacements greater than $\lambda/2$ is restricted to counting fringes if only a single wavelength is used. Fringe counting during measurement of displacement means that any interruption to the measurement or any sudden changes in displacement of $> \lambda/2$ may introduce an error in the measured displacement. As many applications introduce this type of interruption, a multi-wavelength technique has been developed [1-4].

Equation (2) also highlights the need to know accurately the value of the wavelength $\lambda$ of the source for the interferometric measurement. All measurements of $S$ are referenced to $\lambda$ and hence can be no more accurate than $\lambda$. The vacuum wavelength $\lambda_v = \lambda_0$. The refractive index of air $n$ is a function of, $inter alia$, wavelength, air pressure, temperature, concentration of CO$_2$ and humidity [10]. Hence, changes in the refractive index $n$ will affect the accuracy of the measurement of $S$ unless the changes are accounted for or unless $\lambda$ is measured for the given conditions. The refractive index at a wavelength of 1.55 $\mu$m, a pressure of 103.250 kPa, a temperature of 20°C, a concentration of CO$_2$ of 450 ppm and a relative humidity of 50% is estimated to be 1.000268148.

Table 1 gives the changes in the refractive index of air for changes in wavelength, pressure, temperature, concentration of CO$_2$ and concentration of water vapour at a wavelength of 1.55 $\mu$m. Changes in refractive index owing to changes in $\lambda$ and atmospheric conditions limit the accuracy to roughly one part per million, unless the changes are taken into account. In this work, the distances were $< 1$ m such that the uncertainty in displacement owing to the uncertainty in $n$ was $< 1$ $\mu$m.

2.2 Multi-wavelength ranging

Measurements at multiple wavelengths can alleviate some of the difficulties associated with fringe counting and allow interruption of the measurement to occur without affecting the result of the measurement [4]. For the case of two wavelengths, the difference of the phase differences for $\lambda_1$ and $\lambda_1$ is of interest and can be written as

$$\Delta \phi = \phi_2 - \phi_1 = 2\left(\frac{2\pi}{\lambda_2} - \frac{2\pi}{\lambda_1}\right) S = \frac{4\pi}{\lambda_2} S$$

(4)

where $\lambda_{12}$ is a synthetic wavelength, which is always larger than either $\lambda_1$ or $\lambda_2$. As the synthetic wavelength is larger than either $\lambda_1$ or $\lambda_2$, the ambiguity distance has been increased. The smaller the difference between the two wavelengths, the larger the synthetic wavelength and, consequently, the larger the ambiguity length, $\lambda_{12}/2$, is. However, the increase in ambiguity distance comes at the price of reduced resolution. The minimum detectable phase difference is directly related to the signal-to-noise ratio (SNR) of the measurement system. Williams [4] showed that the SNR can be improved by increasing the number of measurements to obtain an unambiguous range. Williams also showed that phase difference resolutions of better than $2\pi / 100$ are possible. For the sake of demonstration, in this work we assume the phase difference resolution to be $2\pi/100$. For a two-wavelength system, the resolution will then be $\lambda_{12}/200$, or 1% of the ambiguity distance.

To achieve a sub-micrometre resolution for a two-wavelength source with a mean wavelength ($\lambda_1 + \lambda_2$)/2 of 1.55 $\mu$m requires a wavelength separation of ($\lambda_1 - \lambda_2$)/2 $> 12$ nm assuming that a phase difference of $2\pi/100$ can be detected. Notice that the wavelength separation to achieve sub-micrometre resolution will decrease if a mean wavelength of $< 1.55$ $\mu$m is used. However, for a wavelength separation of 12 nm and a mean wavelength of 1.55 $\mu$m, the ambiguity length is 100 $\mu$m. Thus, with a two-wave- length measurement system it is possible to measure unambiguously large distances but with reduced resolution. It is necessary to use more than two wavelengths to obtain both a high resolution and a large ambiguity length.

Most of the previous multi-wavelength work was done by using different wavelengths from different laser sources and hence the name multiplex wavelength interferometry. In the case that did use a single diode laser [1], the spectral tuning range of the laser was too short to achieve resolution in the sub-micrometre range. We report sub-micron resolution with a single diode laser with a tuning range of $> 100$ nm when operated in a diffractive optical element short external cavity.

The basic principle of wavelength multiplexing is described in [4]. Wavelength multiplexing depends on taking measurements of the phase difference for multiple wavelengths of the source for the interferometer for each unknown displacement. Assume that four measurements are taken at four wavelengths $\lambda_1$, $\lambda_2$, $\lambda_3$ and $\lambda_4$. If $\lambda_1$ is the longest wavelength then it would be considered the reference measurement. The separation between the wavelengths is determined by the targeted absolute distance (i.e. the ambiguity distance) that is to be measured and the resolution that is desired. Subtracting each measurement of the phase difference $\phi$ from the reference measurement at $\lambda_1$ yields the value of $\Delta \phi$. According to (2), $\Delta \phi$ will give the displacement $S$ between the two arms. Careful choice of the differences of wavelengths ($\lambda_1 - \lambda_i$) will lead from a large ambiguity length and low resolution to a large ambiguity length and a high resolution. When the value ($\lambda_1 - \lambda_2$) $\leq 0.1$ nm and ($\lambda_1 + \lambda_2$)/2 $= 1.55$ $\mu$m, then the ambiguity length will be $> 12$ nm, which is equal to a phase difference of $2\pi$. On the other hand, the resolution will be low which can be improved by calculating a second phase difference by using ($\lambda_3 - \lambda_4$) equal to a couple of nanometres. The only factor that determines the

Table 1: Changes in the refractive index of air at 1.55 $\mu$m owing to changes in wavelength, pressure, temperature, concentration of CO$_2$ and concentration of water vapour

<table>
<thead>
<tr>
<th>Independent variable</th>
<th>$\Delta$ independent variable</th>
<th>$\Delta n$ variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure (101325 kPa)</td>
<td>$1$ kPa</td>
<td>$2.65 \times 10^{-6}$</td>
</tr>
<tr>
<td>Temperature (273.15 K)</td>
<td>$1$ K</td>
<td>$-9.17 \times 10^{-7}$</td>
</tr>
<tr>
<td>Wavelength (1.55 $\mu$m)</td>
<td>$10$ nm</td>
<td>$-8.18 \times 10^{-9}$</td>
</tr>
<tr>
<td>CO$_2$ (450.15 ppm)</td>
<td>$50$ ppm</td>
<td>$7.09 \times 10^{-9}$</td>
</tr>
<tr>
<td>Relative humidity (0.5)</td>
<td>$0.01$</td>
<td>$-8.73 \times 10^{-9}$</td>
</tr>
</tbody>
</table>
value \((\lambda_1 - \lambda_2)\) is the resolution of the previous measurement. The ambiguity length of the second measurement should be slightly larger than the resolution of the first measurement. By doing so, the overall measurement will have the same resolution as the last measurement, which should be better than the resolution of the first measurement. To increase further the resolution, a third phase difference by using the data from \((\lambda_1 - \lambda_3)\) is needed. Again the only factor that determines the value of \((\lambda_1 - \lambda_3)\) is the last resolution achieved. By choosing a wavelength difference of 100 nm for \((\lambda_1 - \lambda_3)\), the overall measurement of distance will maintain the original long ambiguity length and will preserve the latest resolution, which will be better than 0.3 µm. In short, the wavelength multiplexing method will achieve a long ambiguity length determined by the tuning range of the single diode laser. Once all these calculations are completed, one data point for one object position with high resolution will be established. As a result, the wavelength multiplexing method will be able to determine the absolute displacement and the only limits to this method are the range of wavelengths that are available and the accuracy to which the phase difference can be determined.

In short, the goal of the wavelength multiplex method is to pick the wavelengths such that only one solution to the multi-valued inversion of (2) is available.

### 2.3 Cosine fitting method

Another approach to determining the displacement \(S\), and one which we report here, is to fit a harmonic function to the data and to use a best fit parameter to estimate \(S\). It appears that the best fit method allows for extraction of meaningful estimates of \(S\) in the presence of noise. A plot of the output of the interferometer as a function of wavelength shows the characteristic fringes with period equal to the displacement \(S\). With the broadly tunable laser that we employ in the DOE SXC, we can obtain measurements over a broad range of wavelengths and hence measure the output as a function of wavelength.

The displacement \(S\) was extracted from the data by using a nonlinear Marquardt [11] algorithm to minimise \(\chi^2\)

\[
\chi^2 = \sum_{i=1}^{N} \left( y_i - y(\lambda_i) \right)^2
\]

where \(y_i\) are the measured outputs of the interferometer for a monochromatic input at a wavelength of \(\lambda_i\) and \(y(\lambda_i)\) are the values of the nonlinear function calculated as

\[
y(\lambda) = a_1 + a_6(\lambda - 1500) + [a_2 + a_5(\lambda - 1500)]\cos\left(\frac{4\pi a_3}{\lambda} + a_4\right)
\]

The uncertainty for a single measurement was not independently estimated. This uncertainty was estimated from the variance of the fit as the minimum value of \(\chi^2/(N-m)\) where \(N\) was the total number of data points and \(m\) was the number of fit parameters [12]. The uncertainty of the displacement that was extracted from the data was taken as twice the square root of the product of the value of the appropriate element of the error matrix and the variance of the fit \(\chi^2/(N-m)\). The factor of two gives a confidence level of approximately 95%, assuming that the uncertainties followed a normal distribution.

The DC offset and amplitude of the harmonic function were allowed to be functions of wavelength to account for any wavelength dependence in the optical components of the interferometer. The inclusion of these terms improved the fits in that the values of \(\chi^2\) were reduced and the visual qualities of the fits were improved. The inclusion of these terms reduced the number of degrees of freedom for the fit and these terms could be excluded at a loss of accuracy in the determination of \(S\). If these terms were not included in the fit, the accuracy in the determination of the displacement \(S\) changed from 0.05 µm to 0.35 µm when the terms were excluded.

The Whittaker-Shannon sampling theorem can be used to determine the minimum number of points required to make an unambiguous measurement of \(S\) [13,14]. Provided that there are slightly more than two samples per period of the harmonic function, that is, provided that the sampling rate is slightly higher than the Nyquist rate, then it should be possible to recover \(S\) from measurements made at a finite number of wavelengths. The resolution with which \(S\) can be determined will depend on the total number of measurements that are made. This point is demonstrated with the experimental data by fitting to every \(n\)th data point.

The sensitivity of the fit to changes in the displacement \(S\) is demonstrated in Fig. 1. Fig. 1 shows simulated data for displacements of the interferometer arms of 30.0, 30.1, 30.2 and 30.3 µm over a range of wavelengths of 100 nm, a range of wavelengths that can be obtained with the DOE SXC laser. Fig. 1 clearly shows that a 0.1 µm change in the displacement presents a discernable change in the output of the interferometer as a function of wavelength.

### 3 Experimental set-up

The performance of multi-wavelength absolute laser ranging using single laser sources was investigated by using a custom-designed and fabricated (wafer #4381) broadly tunable InGaAsP/InP diode laser [5, 6]. The custom-designed laser was used with uncoated facets and has a compositional AMQW active region that consists of five compressively strained, 100-A˚ thick quantum wells. This custom-designed laser can be tuned in excess of 100 nm, which is needed for this application. Conventional uncoated multiple quantum well (MQW) lasers, where the QWs are nominally of the same composition and thickness, can be tuned over only 40–50 nm [7, 8] under the same conditions as the custom-designed AMQW lasers. The laser was designed [15] and fabricated using the growth and fabrication facilities at McMaster University. The diode laser was tuned by a DOE SXC as described in [7]. The DOE SXC allowed single longitudinal mode operation on each mode within...
the >100 nm tuning range of the lasers. The modes were not necessarily measured in a sequential fashion and as a result typically four measurements of the output of the interferometer were made for each wavelength corresponding to a given longitudinal mode of the diode laser. The cavity length of the diode laser was 850 μm, which gives a mode spacing of 0.5 nm.

A schematic diagram of the experimental set-up is shown in Fig. 2. Selection of the mode of operation of the DOE SXC laser, and hence the wavelength of operation of the distance measuring interferometer, was achieved by moving the DOE along the optical axis of the laser using a BEI linear actuator [LA10-12-027A] in a flexure mount. The output of the diode laser was collimated with a lens (hence the interferometer was more of a Twyman-Green interferometer than a Michelson interferometer [16]) and passed through a beam splitter (BS1). Part of the beam went to an HP86120C multi-wavelength meter, which has an accuracy of ±2 ppm (±0.003 nm) for the wavelengths that were used. The rest of the collimated beam passed through a second beam splitter (BS2) where part of it went to a reference detector and the remainder went to the nominally 50/50 beam splitter (BS3) of the interferometer. The collimated beam was split by BS3 into the two arms of the interferometer where the reference beam went to the reference mirror (M2) and the signal beam went to the object. The object could be moved in three orthogonal directions using a three-dimensional translator stage. Both of the beams were reflected from reference and object mirrors and were combined at the fringe detector. A chopper was inserted before the fringe detector and referenced a lock-in amplifier for phase-sensitive detection. The reference detector and the output of the lock-in amplifier were connected to a HP-54600A digital oscilloscope and to a computer for data acquisition. The temperature of the laser was controlled by a thermo-electric cooling stage. A path length difference S in the interferometer arms was introduced by moving mirror M1. The distance that M1 was moved was recorded with a Oriel Motor-Mike controller [18008] display and by counting fringes from the diode laser. The fringes were counted manually and have an estimated uncertainty of one fringe.

4 Results and discussion

Data from two experiments were taken to verify both the phase subtracting and the cosine fitting methods. The first experiment yielded poor estimates of the displacement when using the phase subtracting method owing to the noisy signals. Consequently, the accuracy of the measurements of displacement using the phase subtraction signal was several micrometers.

The second experiment was done using the same DOE SXC AMQW diode laser. However, this time measurements were made over a greater number of modes, and the data were analysed using a least squares fit of a harmonic function to the data. Fig. 3 shows the output of the interferometer as a function of wavelength for a displacement considered to be the zero position and the best fit line to the data. The figure clearly shows a noisy harmonic signal. The noise is the reason for the poor accuracy that was obtained when the data was analysed by the method of phase difference. The raw data does show the harmonic function nature of the wavelength dependence. This permits the displacement S to be extracted by fitting a harmonic function to the data. The parameter S was extracted through a non-linear least squares fit, yielding 23.4 ± 0.05 μm as the displacement S of the interferometer arms.

To determine the number of raw data points that are required to determine S and the accuracy with which S can be determined, we performed fits using only fractions of the total number of the raw data points. Every 2nd, 4th, 8th, 16th, 32nd, 64th and 128th data points were taken and the function was fit to these sets of data. The extracted displacement S and the resolution for the number of points in the fits are presented in Table 2. A decrease of the resolution to 0.55 μm from 0.05 μm was found as a result of fitting to seven data points instead of all 989 data points. The data in Table 2 shows that fitting to one-eighth of the data points will cause the resolution to decrease to 0.15 μm. Fig. 4 shows samples of the fits for 123, 30, 15 and 7 data points. This is an important result as the number of data points required for a given resolution directly affects the time required to obtain the data.

![Fig. 2 Schematic diagram of the experimental set-up](image)

![Fig. 3 The raw data and the best fit line for all the 989 data points of the scan for a displacement near zero](image)

### Table 2: Extracted displacement S and resolution for different number of data points used in the fit

<table>
<thead>
<tr>
<th># Raw data points used</th>
<th>Extracted S(μm)</th>
<th>Resolution ± (μm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>989</td>
<td>23.39</td>
<td>0.05</td>
</tr>
<tr>
<td>495</td>
<td>23.34</td>
<td>0.07</td>
</tr>
<tr>
<td>247</td>
<td>23.33</td>
<td>0.11</td>
</tr>
<tr>
<td>123</td>
<td>23.42</td>
<td>0.16</td>
</tr>
<tr>
<td>30</td>
<td>23.38</td>
<td>0.28</td>
</tr>
<tr>
<td>15</td>
<td>23.4</td>
<td>0.50</td>
</tr>
<tr>
<td>7</td>
<td>22.99</td>
<td>0.55</td>
</tr>
</tbody>
</table>
The trade-off is higher resolution against a slower measurement of displacement.

Measurements were made for relative displacements of mirror M1 for 20, 40, 60, 80, 100, 200 and 400 \(\mu\text{m}\). The raw data and the best line fits for the raw data are plotted in Fig. 5. The displacements of the arms of the interferometer were extracted from each fit and the difference between the smallest displacement (actuator display = 0) and an extracted displacement yield the measured distance that the mirror was moved. The results from these measurements are presented in Table 3. The column labeled discrepancy is the value of the actuator display minus the measured distance. The measured distance showed sub-micrometer accuracy as compared with the actuator display and was within the estimated experimental uncertainty.

This method can be useful also in other applications rather than distance measurements. For example, it can be used in a surface profiler set-up to give accurate surface topography with better than 0.1-\(\mu\text{m}\) resolution in a non-contact way.

### 5 Conclusion

We have demonstrated a method for measurement of distance with sub-micrometre resolution by using a single, widely tunable diode laser as the source for an interferometer. A custom-designed broadly tunable InGaAsP/InP diode laser was used to demonstrate this method. A DOE SXC was used to tune the laser over 100 nm to achieve measurements of displacement with resolutions as small as 0.05 \(\mu\text{m}\). The method involved a nonlinear, least squares fit to extract the absolute displacements from the raw data. Using this method, we demonstrated that accurate estimates of the absolute displacement can be extracted from raw data even in the presence of noise.

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